Sheet 1

First Order Ordinary Differential Equations

1) Use **<u>Euler's method</u>** to solve $\frac{dy}{dx} = 3x^2 + 1, \quad y(1) = 2$. Take step size = 0.5 and estimate y(2).

2) Solve the equation $\frac{dy}{dx} = x + y, \quad y(0) = 1$ by **<u>Euler's method</u>**. Taking step size = 0.2 and 0.4, find y (1). Compute error in both cases. Compare the results with exact solution ($y = 2e^x - x - 1$).

3) If water is drained from a vertical cylindrical tank by opening a valve at the base, the water will flow fast when the tank is full and slow down as it continues to drain. As turns out, the rate at which the water level drops is:

$$\frac{dh}{dt} = -0.2\sqrt{h}$$

The depth of the water, h, is measured in meter and the time, t, in minute. Determine the depth after 5 minutes if the fluid level is initially 9 m. Solve by applying **<u>Euler's method</u>** use a step of 2.5 minute.

4) The current I in a circuit having a resistance R and inductance L is given by the differential equation as $\frac{dI}{dt} = \frac{E \sin(\Omega t) - RC}{L}$ where E = 100 V, L = 1.5 H, Ω = 500, C = 1 and R = 90 Ohm. Initially, I = 0 at t = 0. By the use of a **<u>midpoint method</u>** find I(0.2) taking time step = 0.1 sec,.

5) Use **<u>midpoint method</u>** to solve $\frac{dy}{dx} = \frac{2y}{x}, \quad y(1) = 2$. Take step size = 0.5 and estimate y(2).

6) A ball at 1200K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only by natural convection with constant-temperature surroundings, the differential equation describing (approximately) the rate of change of temperature T of a ball is given by

$$\frac{dT}{dt} = -0.016(T - 300)$$

Use the **<u>midpoint</u>** method to find the temperature at t=20 seconds. Assume a step size of h=5 seconds

1 - السؤال الاول والرابع محلولين حل نموذجى
2 - السؤال الثانى والخامس سيتم شرحهم فى السكشن
3 - السؤال الثالث والسادس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة

$1 - Given$
$f(x,y) = 3x^2 + 1$
$x_0 = 1$
$y_0 = 2$
$x_f = 2$
$h = 0.5$
$Solution$
y_1
$x_1 = x_0 + h = 1 + 0.5 = 1.5$
$y_1 = y_0 + hf(x_0, y_0) = 2 + 0.5 * f(1,2) = 4$
y_2
$x_2 = x_1 + h = 1.5 + 0.5 = 2$
$y_2 = y_1 + hf(x_1, y_1) = 4 + 0.5 * f(1.5, 4) = 7.875$
y_3
$x_3 = x_2 + h = 2 + 0.5 = 2.5$
$y_3 = y_2 + hf(x_2, y_2) = 7.875 + 0.5 * f(2, 7.875) = 14.375$
y_4
$x_4 = x_3 + h = 2.5 + 0.5 = 3$
$y_4 = y_3 + hf(x_3, y_3) = 14.375 + 0.5 * f(2.5, 14.375) = 24.250$

x	y
1	2
1.5	4
2	7.875
2.5	14.375
3	24.250

4-

$$\frac{\mathrm{dI}}{\mathrm{dt}} = \frac{\mathrm{E}\sin(\Omega \mathrm{t}) - \mathrm{RC}}{\mathrm{L}}$$

substituting constants gives

$$\frac{\mathrm{dI}}{\mathrm{dt}} = (100\sin(500\mathrm{t}) - 90)/1.5$$

let y=I and x=t then

$$\frac{\mathrm{dy}}{\mathrm{dx}} = (100\sin(500\mathrm{x}) - 90)/1.5, \qquad \mathrm{y}(0) = 0$$

Given

$$f(x,y) = \frac{100\,sin(500x) - 90}{1.5}$$

$$x_0 = 0$$

$$y_0 = 0$$

$$x_f = 0.2$$

$$h = 0.1$$

Solution

y_1

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$x_m = x_0 + \frac{h}{2} = 0 + \frac{0.1}{2} = 0.05$$

$$y_m = y_0 + h\frac{f(x_0,y_0)}{2} = 0 + 0.1 * \frac{f(0,0)}{2} = -3$$

$$y_1 = y_0 + hf(x_m,y_m) = 0 + 0.1 * f(0.05,-3) = -6.882$$

y_2

$$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$$

$$x_m = x_1 + \frac{h}{2} = 0.1 + \frac{0.1}{2} = 0.15$$

$$y_m = y_1 + h\frac{f(x_1,y_1)}{2} = -6.882 + 0.1 * \frac{f(0.1,-6.882)}{2} = -10.757$$

$$y_2 = y_1 + hf(x_m,y_m) = -6.882 + 0.1 * f(0.15,-10.757) = -15.468$$

t	I
0.000	0.000
0.100	-6.882
0.200	-15.468